AP Calculus AB — WS 75 - First Derivative Test

1) $f(x) = 3x^2 - 3x + 2$

$f'(x) = 6x - 3$

$6x - 3 = 0$

$x = \frac{1}{2}$

$f'(x)$: $-$ | $\frac{1}{2}$ | $+$

$f(x)$: ↘ $\frac{1}{2}$ ↗

* $f(x)$ has a local min @ $x = \frac{1}{2}$ b/c $f'(x)$ changes signs from $-$ to $+$.

* $f(x)$ is decreasing on $(-\infty, \frac{1}{2})$ b/c $f'(x) < 0$
* $f(x)$ is increasing on $(\frac{1}{2}, \infty)$ b/c $f'(x) > 0$

2) $f(x) = x^3 - x^2 - x$

$f'(x) = 3x^2 - 2x - 1$

$3x^2 - 2x - 1 = 0$

$(3x + 1)(x - 1) = 0$

$x = -\frac{1}{3}$ $\quad x = 1$

$f'(x)$: $+$ | $-\frac{1}{3}$ | $-$ | 1 | $+$

f: ↗ $-\frac{1}{3}$ ↘ 1 ↗

* $f(x)$ is inc on $(-\infty, \frac{1}{3})$ & $(1, \infty)$ b/c $f'(x) > 0$
* $f(x)$ is dec on $(\frac{1}{3}, 1)$ b/c $f'(x) < 0$

* $f(x)$ has a local max @ $x = -\frac{1}{3}$ b/c $f'(x)$ changes signs from $+$ to $-$

* $f(x)$ has a local min @ $x = 1$ b/c $f'(x)$ changes signs from $-$ to $+$.

3) $f(x) = 2x^3 - 9x^2 + 2$

$f'(x) = 6x^2 - 18x = 0$

$6x(x - 3) = 0$

$x = 0$ $\quad x = 3$

$f'(x)$: $+$ | 0 | $-$ | 3 | $+$

f: ↗ 0 ↘ 3 ↗

* $f(x)$ is inc on $(-\infty, 0)$ & $(3, \infty)$ b/c $f'(x) > 0$
* $f(x)$ is dec on $(0, 3)$ b/c $f'(x) < 0$
* $f(x)$ has a local max @ $x = 0$ b/c $f'(x)$ changes signs from $+$ to $-$.
* $f(x)$ has a local min @ $x = 3$ b/c $f'(x)$ changes signs from $-$ to $+$.

4) $f(x) = \frac{1}{4}x^4 - x^3 + x^2$

$f'(x) = x^3 - 3x^2 + 2x = 0$

$x(x^2 - 3x + 2) = 0$

$x(x - 2)(x - 1) = 0$

$x = 0$ $\quad x = 2$ $\quad x = 1$

$f'(x)$: $-$ | 0 | $+$ | 1 | $-$ | 2 | $+$

$f(x)$: ↘ 0 ↗ 1 ↘ 2 ↗

* $f(x)$ is dec on $(-\infty, 0)$ & $(1, 2)$ b/c $f'(x) < 0$
* $f(x)$ is inc on $(0, 1)$ & $(2, \infty)$ b/c $f'(x) > 0$
* $f(x)$ has a local min @ $x = 0$ & $x = 2$ b/c $f'(x)$ Δs signs from $-$ to $+$.
* $f(x)$ has a local max @ $x = 1$ b/c $f'(x)$ Δs signs from $+$ to $-$.

5) $f(x) = \dfrac{x-2}{x+2}$ $\quad x \neq -2$ (vertical asymptote)

$f'(x) = \dfrac{(x+2)-(x-2)}{(x+2)^2}$

$f'(x)$: $+$ $\quad$ -2 $\quad$ $+$

$f'(x) = \dfrac{4}{(x+2)^2}$

No Critical Values

$f(x)$ is increasing on $(-\infty, -2)$ & $(-2, \infty)$ b/c $f'(x) > 0$

6) $f(x) = x^3 + ax^2 + b$

$f(2) = 8 + 4a + b = 3$

$8 + 4a + b = 3$

$8 - 12 + b = 3$

$\boxed{b = 7}$

$f'(x) = 3x^2 + 2ax$

$f'(2) = 12 + 4a$

$12 + 4a = 0$

$\boxed{a = -3}$

7) $f'(x) = x(x-3)^2(x+1)^4 = 0$

$x = 0 \quad x = 3 \quad x = -1$

$f'(x)$: $-$ $\quad$ -1 $\quad$ $-$ $\quad$ 0 $\quad$ $+$ $\quad$ 3 $\quad$ $+$

$\therefore f(x)$ has one local min @ $x = 0$.

8) a) $f' = 0$ @ $x = -1$ & $x = 1$

b) $f' < 0$ on $(-1, 1)$ since $f(x)$ is decreasing.

c) $f' > 0$ on $(-\infty, -1)$ & $(1, \infty)$ b/c $f(x)$ is increasing

9) a) $f' = 0$ @ $x = -1, 0, 1$

b) $f' < 0$ on $(-\infty, -1)$ & $(0, 1)$ b/c $f(x)$ is decreasing

c) $f' > 0$ on $(-1, 0)$ & $(1, \infty)$ b/c $f(x)$ is increasing