

AP Calculus AB

WS 75 - First Derivative Test

1) $f(x) = 3x^2 - 3x + 2$

$$\begin{array}{l} f'(x) \\ \hline - & + \\ f(x) & \downarrow \end{array}$$

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$$f'(x) = 6x - 3$$

$$6x - 3 = 0$$

$x = \frac{1}{2}$ * $f(x)$ has a local min @ $x = \frac{1}{2}$ b/c $f'(x)$ changes signs from - to +

- * $f(x)$ is decreasing on $(-\infty, \frac{1}{2})$ b/c $f'(x) < 0$
- * $f(x)$ is increasing on $(\frac{1}{2}, \infty)$ b/c $f'(x) > 0$.

2) $f(x) = x^3 - x^2 - x$

$$\begin{array}{l} f'(x) = 3x^2 - 2x - 1 \\ 3x^2 - 2x - 1 = 0 \\ (3x + 1)(x - 1) = 0 \\ x = -\frac{1}{3} \quad x = 1 \end{array}$$

$$\begin{array}{l} f'(x) \\ \hline + & - & + \\ f(x) & \uparrow \downarrow \end{array}$$

$$\begin{array}{l} f'(x) \\ \hline + & - & + \\ f(x) & \uparrow \downarrow \end{array}$$

* $f(x)$ is inc on $(-\infty, -\frac{1}{3}) \cup (1, \infty)$ b/c $f'(x) > 0$

* $f(x)$ is dec on $(-\frac{1}{3}, 1)$ b/c $f'(x) < 0$

* $f(x)$ has a local max @ $x = -\frac{1}{3}$ b/c $f'(x)$ changes signs from + to -

* $f(x)$ has a local min @ $x = 1$ b/c $f'(x)$ changes signs from - to +.

3) $f(x) = 2x^3 - 9x^2 + 2$

$$\begin{array}{l} f'(x) = 6x^2 - 18x = 0 \\ 6x(x - 3) = 0 \end{array}$$

$$x = 0 \quad x = 3$$

$$\begin{array}{l} f'(x) \\ \hline + & - & + \\ f(x) & \uparrow \downarrow \end{array}$$

* $f(x)$ is inc on $(-\infty, 0) \cup (3, \infty)$ b/c $f'(x) > 0$

* $f(x)$ is dec on $(0, 3)$ b/c $f'(x) < 0$

* $f(x)$ has a local max @ $x = 0$ b/c $f'(x)$ changes signs from + to -.

* $f(x)$ has a local min @ $x = 3$ b/c $f'(x)$ changes signs from - to +.

4) $f(x) = \frac{1}{4}x^4 - x^3 + x^2$

$$f'(x) = x^3 - 3x^2 + 2x = 0$$

$$x(x^2 - 3x + 2) = 0$$

$$x(x-2)(x-1) = 0$$

$$x = 0 \quad x = 2 \quad x = 1$$

$$\begin{array}{l} f'(x) \\ \hline - & + & - & + \\ f(x) & \downarrow \uparrow \downarrow \uparrow \end{array}$$

* $f(x)$ is dec on $(-\infty, 0) \cup (1, 2)$ b/c $f'(x) < 0$

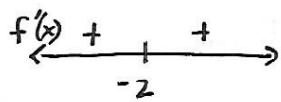
* $f(x)$ is inc on $(0, 1) \cup (2, \infty)$ b/c $f'(x) > 0$

* $f(x)$ has a local min @ $x = 0 \neq x = 2$ b/c $f'(x)$ changes signs from - to +.

* $f(x)$ has a local max @ $x = 1$ b/c $f'(x)$ changes signs from + to -.

5) $f(x) = \frac{x-2}{x+2}$ $x \neq -2$ (vertical asymptote)

$$f'(x) = \frac{(x+2)-(x-2)}{(x+2)^2}$$



$$f'(x) = \frac{4}{(x+2)^2}$$

No Critical Values

$f(x)$ is increasing on $(-\infty, -2) \cup (-2, \infty)$ b/c $f'(x) > 0$

c) $f(x) = x^3 + ax^2 + b$ $f'(x) = 3x^2 + 2ax$

$$f(2) = 8 + 4a + b = 3 \quad f'(2) = 12 + 4a$$

$$8 + 4a + b = 3$$

$$8 - 12 + b = 3$$

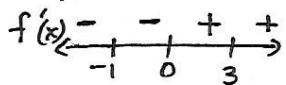
$$\boxed{b = 7}$$

$$12 + 4a = 0$$

$$\boxed{a = -3}$$

7) $f'(x) = x(x-3)^2(x+1)^4 = 0$

$$x=0 \quad x=3 \quad x=-1$$



$\therefore f(x)$ has one local min

$$\textcircled{Q} \quad x=0.$$

8) a) $f' = 0$ @ $x = -1$ & $x = 1$

b) $f' < 0$ on $(-1, 1)$ since

$f(x)$ is decreasing.

c) $f' > 0$ on $(-\infty, -1) \cup (1, \infty)$ b/c

$f(x)$ is increasing

9) a) $f' = 0$ @ $x = -1, 0, 1$

b) $f' < 0$ on $(-\infty, -1) \cup (0, 1)$ b/c

$f(x)$ is decreasing

c) $f' > 0$ on $(-1, 0) \cup (1, \infty)$ b/c

$f(x)$ is increasing